Beispiele zu polynomiellen Reduktionen

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9. Februar 2011

 $SAT \leq_p 3-SAT$

3-SAT \leq_p CLIQUE

 $3-SAT \leq_p EXACT-ONE 3-SAT$

EXACT-ONE 3-SAT \leq_p SUBSET SUM

SUBSET SUM \leq_p PARTITION

 $SAT \leq_p DHP$

 $\mathsf{DHP} \leq_{p} \mathsf{HP}$

 $HP \leq_p START-ZIEL HP$

$$\textit{C} = \left(\textit{x}_{1} \lor \overline{\textit{x}_{4}}\right) \land \left(\overline{\textit{x}_{1}} \lor \textit{x}_{2} \lor \textit{x}_{3} \lor \overline{\textit{x}_{4}} \lor \textit{x}_{5}\right) \land \left(\textit{x}_{1} \lor \overline{\textit{x}_{2}} \lor \overline{\textit{x}_{3}} \lor \overline{\textit{x}_{4}}\right) \land \overline{\textit{x}_{3}}$$

$$C = (x_1 \vee \overline{x_4}) \wedge (\overline{x_1} \vee x_2 \vee x_3 \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3} \vee \overline{x_4}) \wedge \overline{x_3}$$

$$f(C) = (x_1 \vee \overline{x_4} \vee a) \wedge (x_1 \vee \overline{x_4} \vee \overline{a}) \wedge (\overline{x_1} \vee x_2 \vee h_2) \wedge (\overline{h_2} \vee x_3 \vee h_3) \wedge (\overline{h_3} \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee h_2') \wedge (\overline{h_2'} \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_3} \vee b \vee c) \wedge (\overline{x_3} \vee \overline{b} \vee c) \wedge (\overline{x_3} \vee b \vee \overline{c}) \wedge (\overline{x_3} \vee \overline{b} \vee \overline{c})$$

$$C = (x_1 \vee \overline{x_4}) \wedge (\overline{x_1} \vee x_2 \vee x_3 \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3} \vee \overline{x_4}) \wedge \overline{x_3}$$

$$f(C) = (x_1 \vee \overline{x_4} \vee a) \wedge (x_1 \vee \overline{x_4} \vee \overline{a}) \wedge (\overline{x_1} \vee x_2 \vee h_2) \wedge (\overline{h_2} \vee x_3 \vee h_3) \wedge (\overline{h_3} \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee h_2') \wedge (\overline{h_2'} \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_3} \vee b \vee c) \wedge (\overline{x_3} \vee \overline{b} \vee c) \wedge (\overline{x_3} \vee b \vee \overline{c}) \wedge (\overline{x_3} \vee \overline{b} \vee \overline{c})$$

$$C = (x_1 \vee \overline{x_4}) \wedge (\overline{x_1} \vee x_2 \vee x_3 \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3} \vee \overline{x_4}) \wedge \overline{x_3}$$

$$f(C) = (x_1 \vee \overline{x_4} \vee a) \wedge (x_1 \vee \overline{x_4} \vee \overline{a}) \wedge (\overline{x_1} \vee x_2 \vee h_2) \wedge (\overline{h_2} \vee x_3 \vee h_3) \wedge (\overline{h_3} \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee h_2') \wedge (\overline{h_2'} \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_3} \vee b \vee c) \wedge (\overline{x_3} \vee \overline{b} \vee c) \wedge (\overline{x_3} \vee b \vee \overline{c}) \wedge (\overline{x_3} \vee \overline{b} \vee \overline{c})$$

$$C = (x_1 \vee \overline{x_4}) \wedge (\overline{x_1} \vee x_2 \vee x_3 \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3} \vee \overline{x_4}) \wedge \overline{x_3}$$

$$f(C) = (x_1 \vee \overline{x_4} \vee a) \wedge (x_1 \vee \overline{x_4} \vee \overline{a}) \wedge (\overline{x_1} \vee x_2 \vee h_2) \wedge (\overline{h_2} \vee x_3 \vee h_3) \wedge (\overline{h_3} \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee h_2') \wedge (\overline{h_2'} \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_3} \vee b \vee c) \wedge (\overline{x_3} \vee \overline{b} \vee c) \wedge (\overline{x_3} \vee b \vee \overline{c}) \wedge (\overline{x_3} \vee \overline{b} \vee \overline{c})$$

$$C = (x_1 \vee \overline{x_4}) \wedge (\overline{x_1} \vee x_2 \vee x_3 \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3} \vee \overline{x_4}) \wedge \overline{x_3}$$

$$f(C) = (x_1 \vee \overline{x_4} \vee a) \wedge (x_1 \vee \overline{x_4} \vee \overline{a}) \wedge (\overline{x_1} \vee x_2 \vee h_2) \wedge (\overline{h_2} \vee x_3 \vee h_3) \wedge (\overline{h_3} \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee h_2') \wedge (\overline{h_2'} \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_3} \vee b \vee c) \wedge (\overline{x_3} \vee \overline{b} \vee c) \wedge (\overline{x_3} \vee \overline{b} \vee \overline{c})$$

$$C = (x_1 \vee \overline{x_4}) \wedge (\overline{x_1} \vee x_2 \vee x_3 \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3} \vee \overline{x_4}) \wedge \overline{x_3}$$

$$f(C) = (x_1 \vee \overline{x_4} \vee a) \wedge (x_1 \vee \overline{x_4} \vee \overline{a}) \wedge (\overline{x_1} \vee x_2 \vee h_2) \wedge (\overline{h_2} \vee x_3 \vee h_3) \wedge (\overline{h_3} \vee \overline{x_4} \vee x_5) \wedge (x_1 \vee \overline{x_2} \vee h_2') \wedge (\overline{h_2'} \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_3} \vee b \vee c) \wedge (\overline{x_3} \vee \overline{b} \vee c) \wedge (\overline{x_3} \vee b \vee \overline{c}) \wedge (\overline{x_3} \vee \overline{b} \vee \overline{c})$$

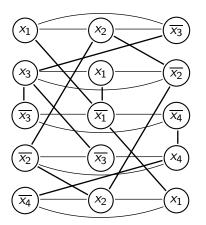
Reduktionsabbildung von 3-SAT auf CLIQUE

$$(x_1 \lor x_2 \lor \overline{x_3}) \land (x_3 \lor x_1 \lor \overline{x_2}) \land (\overline{x_3} \lor \overline{x_1} \lor \overline{x_4}) \land (\overline{x_2} \lor \overline{x_3} \lor x_4) \land (\overline{x_4} \lor x_2 \lor x_1)$$

Reduktionsabbildung von 3-SAT auf CLIQUE

$$(x_1 \lor x_2 \lor \overline{x_3}) \land (x_3 \lor x_1 \lor \overline{x_2}) \land (\overline{x_3} \lor \overline{x_1} \lor \overline{x_4}) \land (\overline{x_2} \lor \overline{x_3} \lor x_4) \land (\overline{x_4} \lor x_2 \lor x_1)$$

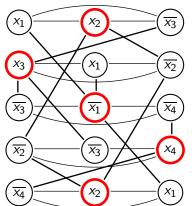
Komplementgraph (Eine Clique erscheint hier als paarweise unverbundene Knoten)



Reduktionsabbildung von 3-SAT auf CLIQUE

$$(x_1 \lor x_2 \lor \overline{x_3}) \land (x_3 \lor x_1 \lor \overline{x_2}) \land (\overline{x_3} \lor \overline{x_1} \lor \overline{x_4}) \land (\overline{x_2} \lor \overline{x_3} \lor x_4) \land (\overline{x_4} \lor x_2 \lor x_1)$$

Erfüllende Belegung: $x_1 = 0, x_2 = 1, x_3 = 1$ und $x_4 = 1$
4-Clique



$$C = (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_4})$$

$$C = (x_{1} \lor \overline{x_{2}} \lor x_{3}) \land (x_{1} \lor \overline{x_{2}} \lor \overline{x_{4}})$$

$$f(C) = (g_{0} \lor g \lor g') \land (g_{0} \lor \overline{g} \lor \overline{g'}) \land (b_{1,1} \lor b_{1,2} \lor b_{1,3}) \land (x_{1} \lor h_{1,1} \lor b'_{1,1}) \land (\overline{x_{2}} \lor h_{1,2} \lor g_{0}) \land (b_{1,2} \lor h_{1,2} \lor h'_{1,2}) \land (x_{3} \lor h_{1,3} \lor g_{0}) \land (b_{1,3} \lor h_{1,3} \lor h'_{1,3}) \land (b_{2,1} \lor b_{2,2} \lor b_{2,3}) \land (x_{1} \lor h_{2,1} \lor g_{0}) \land (b_{2,1} \lor h_{2,1} \lor h'_{2,1}) \land (\overline{x_{2}} \lor h_{2,2} \lor g_{0}) \land (b_{2,2} \lor h_{2,2} \lor h'_{2,2}) \land (\overline{x_{4}} \lor h_{2,3} \lor g_{0}) \land (b_{2,3} \lor h_{2,3} \lor h'_{2,3})$$

$$C = (x_{1} \lor \overline{x_{2}} \lor x_{3}) \land (x_{1} \lor \overline{x_{2}} \lor \overline{x_{4}})$$

$$f(C) = (g_{0} \lor g \lor g') \land (g_{0} \lor \overline{g} \lor \overline{g'}) \land (b_{1,1} \lor b_{1,2} \lor b_{1,3}) \land (x_{1} \lor h_{1,1} \lor b'_{1,1}) \land (\overline{x_{2}} \lor h_{1,2} \lor g_{0}) \land (b_{1,1} \lor h_{1,1} \lor h'_{1,1}) \land (\overline{x_{2}} \lor h_{1,2} \lor g_{0}) \land (b_{1,2} \lor h_{1,2} \lor h'_{1,2}) \land (x_{3} \lor h_{1,3} \lor g_{0}) \land (b_{1,3} \lor h_{1,3} \lor h'_{1,3}) \land (b_{2,1} \lor b_{2,2} \lor b_{2,3}) \land (x_{1} \lor h_{2,1} \lor g_{0}) \land (b_{2,1} \lor h_{2,1} \lor h'_{2,1}) \land (\overline{x_{2}} \lor h_{2,2} \lor g_{0}) \land (b_{2,2} \lor h_{2,2} \lor h'_{2,2}) \land (\overline{x_{4}} \lor h_{2,3} \lor g_{0}) \land (b_{2,3} \lor h_{2,3} \lor h'_{2,3})$$

$$C = (x_{1} \lor \overline{x_{2}} \lor x_{3}) \land (x_{1} \lor \overline{x_{2}} \lor \overline{x_{4}})$$

$$f(C) = (g_{0} \lor g \lor g') \land (g_{0} \lor \overline{g} \lor \overline{g'}) \land (b_{1,1} \lor b_{1,2} \lor b_{1,3}) \land (x_{1} \lor h_{1,1} \lor g_{0}) \land (b_{1,1} \lor h_{1,1} \lor h'_{1,1}) \land (\overline{x_{2}} \lor h_{1,2} \lor g_{0}) \land (b_{1,2} \lor h_{1,2} \lor h'_{1,2}) \land (x_{3} \lor h_{1,3} \lor g_{0}) \land (b_{1,3} \lor h_{1,3} \lor h'_{1,3}) \land (b_{2,1} \lor b_{2,2} \lor b_{2,3}) \land (x_{1} \lor h_{2,1} \lor g_{0}) \land (b_{2,1} \lor h_{2,1} \lor h'_{2,1}) \land (\overline{x_{2}} \lor h_{2,2} \lor g_{0}) \land (b_{2,2} \lor h_{2,2} \lor h'_{2,2}) \land (\overline{x_{4}} \lor h_{2,3} \lor g_{0}) \land (b_{2,3} \lor h_{2,3} \lor h'_{2,3})$$

Reduktionsabb. von EXACT-ONE 3-SAT auf SUBSET SUM $(\overline{x_1} \lor x_2 \lor \overline{x_3}) \land (\overline{x_1} \lor \overline{x_2} \lor x_3) \land (x_1 \lor x_3 \lor \overline{x_4}) \land (\overline{x_2} \lor x_3 \lor \overline{x_4}) \land (\overline{x_1} \lor x_2 \lor x_4)$

$$\big(\overline{x_1} \lor x_2 \lor \overline{x_3}\big) \land \big(\overline{x_1} \lor \overline{x_2} \lor x_3\big) \land \big(x_1 \lor x_3 \lor \overline{x_4}\big) \land \big(\overline{x_2} \lor x_3 \lor \overline{x_4}\big) \land \big(\overline{x_1} \lor x_2 \lor x_4\big)$$

$$S_1 = \{3\}, S_2 = \{1, 5\}, S_3 = \{2, 3, 4\}, S_4 = \{5\}$$

 $S_5 = \{1, 2, 5\}, S_6 = \{2, 4\}, S_7 = \{1\}, S_8 = \{3, 4\}$

$$\big(\overline{x_1} \lor x_2 \lor \overline{x_3}\big) \land \big(\overline{x_1} \lor \overline{x_2} \lor x_3\big) \land \big(x_1 \lor x_3 \lor \overline{x_4}\big) \land \big(\overline{x_2} \lor x_3 \lor \overline{x_4}\big) \land \big(\overline{x_1} \lor x_2 \lor x_4\big)$$

Erfüllende Belegung: $x_1 = 1, x_2 = 0, x_3 = 0$ und $x_4 = 1$ entspricht der Summe von $a_1 + a_4 + a_6 + a_7 = S$

$$a_{1} = (8^{3}) + (8^{6})$$

$$a_{2} = (8^{1} +8^{5}) + (8^{7})$$

$$a_{3} = (8^{2} +8^{3} +8^{4}) + (8^{8})$$

$$a_{4} = (8^{5}) + (8^{9})$$

$$a_{5} = (8^{1} 8^{2} +8^{5}) + (8^{6})$$

$$a_{6} = (8^{2} +8^{4}) + (8^{7})$$

$$a_{7} = (8^{1}) + (8^{8})$$

$$a_{8} = (8^{3} +8^{4}) + (8^{9})$$

$$S = (8^{1} +8^{2} +8^{3} +8^{4} +8^{5}) + (8^{6} +8^{7} +8^{8} +8^{9})$$

$$(\overline{x_1} \lor x_2 \lor \overline{x_3}) \land (\overline{x_1} \lor \overline{x_2} \lor x_3) \land (\underline{x_1} \lor x_3 \lor \overline{x_4}) \land (\overline{x_2} \lor x_3 \lor \overline{x_4}) \land (\overline{x_1} \lor x_2 \lor \underline{x_4})$$

Erfüllende Belegung: $x_1 = 1, x_2 = 0, x_3 = 0$ und $x_4 = 1$ entspricht der Summe von $a_1 + a_4 + a_6 + a_7 = S$

$$a_{1} = (8^{3}) + (8^{6})$$

$$a_{2} = (8^{1} + 8^{3} + 8^{4}) + (8^{7})$$

$$a_{3} = (8^{2} + 8^{3} + 8^{4}) + (8^{8})$$

$$a_{4} = (8^{5}) + (8^{9})$$

$$a_{5} = (8^{1} + 8^{2} + 8^{4}) + (8^{7})$$

$$a_{6} = (8^{2} + 8^{4}) + (8^{7})$$

$$a_{7} = (8^{1} + 8^{2} + 8^{3} + 8^{4}) + (8^{9})$$

$$S = (8^{1} + 8^{2} + 8^{3} + 8^{4} + 8^{5}) + (8^{6} + 8^{7} + 8^{8} + 8^{9})$$

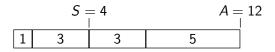
Reduktionsabbildung von SUBSET SUM auf PARTITION

Eingabe von SUBSET SUM: $\{1,3,3,5,4\}$

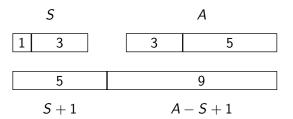
S=4			<i>A</i> =	= 12
1	3	3	5	

Reduktionsabbildung von SUBSET SUM auf PARTITION

Eingabe von SUBSET SUM: $\{1,3,3,5,4\}$



Zugehörige Eingabe von PARTITION : $\{1,3,3,5,5,9\}$



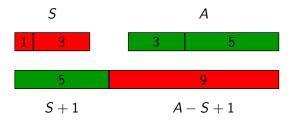
Reduktionsabbildung von SUBSET SUM auf PARTITION

Lösung

Eingabe von SUBSET SUM: $\{1,3,3,5,4\}$

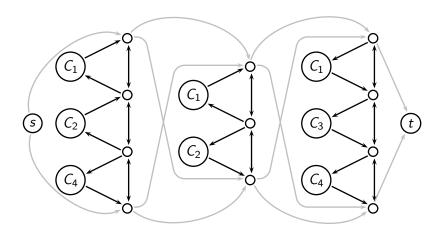


Zugehörige Eingabe von PARTITION : $\{1,3,3,5,5,9\}$



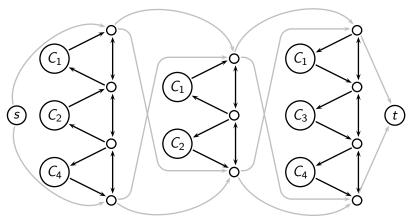
$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$

$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$



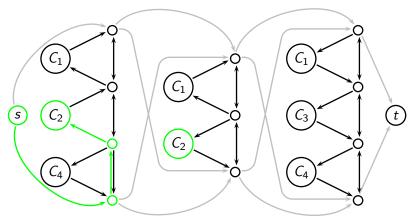
$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$

Erfüllende Belegung: $x_1 = 0, x_2 = 1, x_3 = 1$



$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$

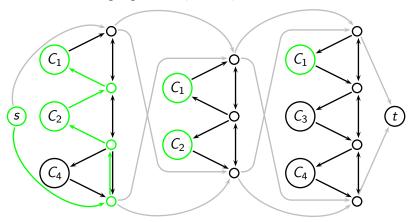
Erfüllende Belegung: $x_1 = 0, x_2 = 1, x_3 = 1$





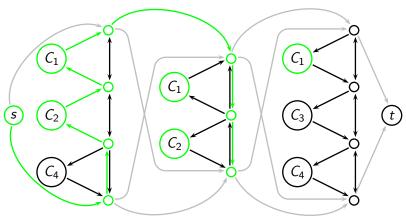
$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$

Erfüllende Belegung: $x_1 = 0, x_2 = 1, x_3 = 1$



$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$

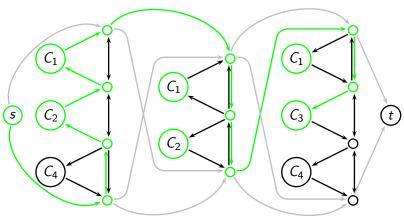
Erfüllende Belegung: $x_1 = 0, x_2 = 1, x_3 = 1$





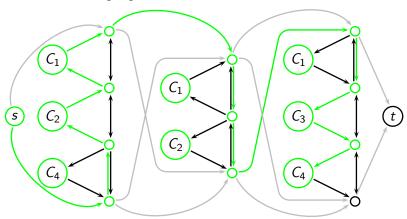
$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$

Erfüllende Belegung: $x_1 = 0, x_2 = 1, x_3 = 1$



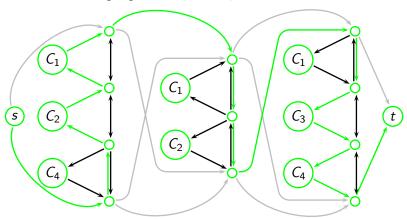
$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$

Erfüllende Belegung: $x_1 = 0, x_2 = 1, x_3 = 1$

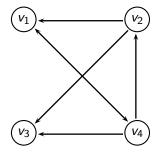


$$C = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2) \wedge x_3 \wedge (x_1 \vee x_3)$$

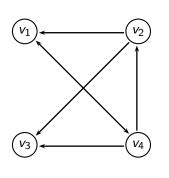
Erfüllende Belegung: $x_1 = 0, x_2 = 1, x_3 = 1$

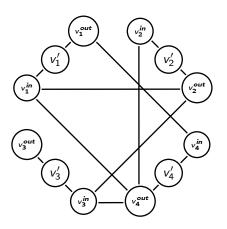


DHP



DHP HP

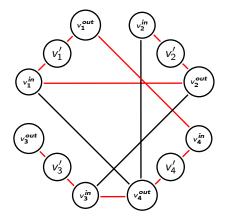




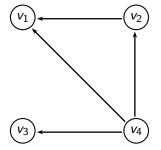
*V*4

DHP Lösung in DHP HP

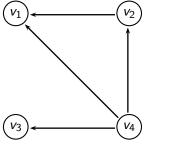
Lösung in HP

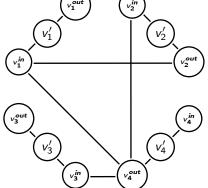


DHP

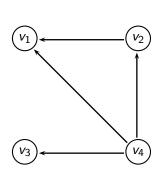


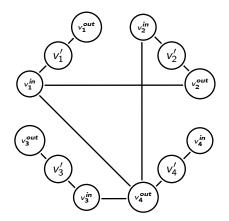
DHP HP v_1 v_2 v_1 v_2 v_2 v_2 v_2 v_2 v_2



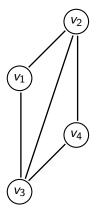


DHP HP Keine Lösung möglich

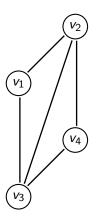




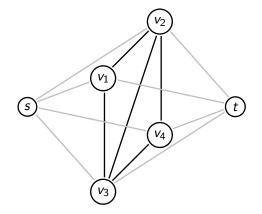
ΗP



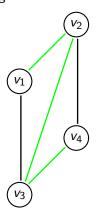
HP STA



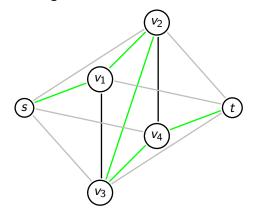




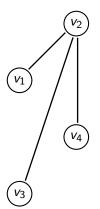
HP Lösung in HP



START-ZIEL HP Lösung in START-ZIEL HP

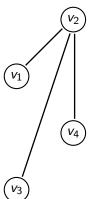


ΗP



ΗP START-ZIEL HP (s)

HP Keine Lösung möglich



START-ZIEL HP

